Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Wednesday 15 May 2019

Morning (Time: 2 hours) | Paper Reference **8MA0/01**

Mathematics

Advanced Subsidiary

Paper 1: Pure Mathematics

Model Solutions

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

3/1/1/1/

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Pearson

1. The line l_1 has equation $2x + 4y - 3 = 0$

The line l_2 has equation $y = mx + 7$, where m is a constant.

Given that l_1 and l_2 are perpendicular,

(a) find the value of m.

(2)

The lines l_1 and l_2 meet at the point P.

(b) Find the x coordinate of P.

(2)

a) $l_1: \; 4y = -2x + 3$

$\div 4: \; y = -\frac{2x}{4} + \frac{3}{4}$

so $m_{l_1} = -\frac{1}{2}$ //

l_1 and l_2 are perpendicular:

$-\frac{1}{2} \times m = -1$

$\therefore m = \frac{-1}{-\frac{1}{2}} = \boxed{2}$

b) $y = \underbrace{2x + 7}_{l_2} = \underbrace{-\frac{1}{2}x + \frac{3}{4}}_{l_1}$

$\Rightarrow \frac{5}{2}x = -\frac{25}{4}$

$\Rightarrow \boxed{x = -\frac{5}{2}}$

2. Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

i) $16a^2 - 2\sqrt{a} = 0$

$a^{\frac{1}{2}}\left[16a^{3/2} - 2\right] = 0$

so $a^{\frac{1}{2}} = 0$ or $16a^{3/2} - 2 = 0$

$\Rightarrow \boxed{a = 0}$

$a^{3/2} = \frac{1}{8}$

$a^{1/2} = \frac{1}{2}$

$\Rightarrow \boxed{a = \frac{1}{4}}$

ii) let $b^2 = y$,

$y^2 + 7y - 18 = 0$

$(y+9)(y-2) = 0$

$\Rightarrow y = -9$ $y = 2$

$b^2 = -9$ $b^2 = 2$

[no real solutions] $\boxed{b = \pm\sqrt{2}}$

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3. (a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) \mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) \mathrm{d}x = 8$$

(3)

a) $\int [4x^{-3} + kx] dx = \frac{4x^{-2}}{-2} + \frac{kx^2}{2} + c$

$= \boxed{\frac{-2}{x^2} + \frac{k}{2}x^2 + c}$

b) $\left[\frac{-2}{x^2} + \frac{k}{2}x^2 \right]_{\frac{1}{2}}^{2} = 8$

$\left[\frac{-2}{4} + \frac{4k}{2} \right] - \left[\frac{-2}{\frac{1}{4}} + \frac{k}{2} \cdot \frac{1}{4} \right] = 8$

$\left[-\frac{1}{2} + \cancel{8} \right] + \left[2k - \frac{k}{8} \right] = \cancel{8}$

$\frac{15}{8}k = \frac{1}{2} \quad \therefore \boxed{k = \frac{4}{15}}$

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Question 3 continued

(Total for Question 3 is 6 marks)

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4. A tree was planted in the ground.
Its height, H metres, was measured t years after planting.

Exactly 3 years after planting, the height of the tree was 2.35 metres.
Exactly 6 years after planting, the height of the tree was 3.28 metres.

Using a linear model,

(a) find an equation linking H with t.

(3)

The height of the tree was approximately 140 cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a).

(2)

a) $t=3,\ H=2.35$

$t=6,\ H=3.28$

gradient $= \dfrac{H_2-H_1}{t_2-t_1} = \dfrac{3.28-2.35}{6-3}$

$= 0.31$ //

so ... $H-H_0=m(t-t_0)$

$H-2.35=0.31(t-3)$

$\boxed{H=0.31t+1.42}$

b) $\underline{t=0}$: $H=0.31(0)+1.42 = 1.42\text{m}$
$= 142\text{cm}$

from our model we can see at $t=0$, $H=142 \simeq 140$.

So this fact does support our linear model!

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5. A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the exact range of values of x for which the curve is increasing.

(2)

a) $y = 3x^2 + 24x^{-1} + 2$

$\frac{dy}{dx} = 6x - 24x^{-2} = \boxed{6x - \frac{24}{x^2}}$

b) curve is increasing; $\frac{dy}{dx} > 0$

$\Rightarrow \quad 6x - \frac{24}{x^2} > 0$

$\Rightarrow \quad 6x^3 - 24 > 0$

$\Rightarrow \quad x^3 > 4$

$\Rightarrow \quad \boxed{x > 4^{\frac{1}{3}}}$

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6.

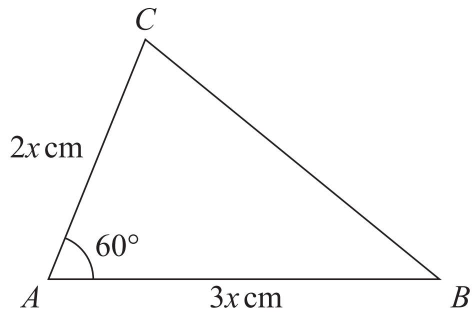


Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle ABC is $18\sqrt{3}$ cm^2

(a) show that $x = 2\sqrt{3}$

(3)

(b) Hence find the exact length of BC, giving your answer as a simplified surd.

(3)

a) $\text{Area} = \frac{1}{2}ab\sin C = 18\sqrt{3}$

$\Rightarrow \frac{1}{2}(2x)(3x)\sin 60 = 18\sqrt{3}$

$\Rightarrow 3x^2 \cdot \frac{\cancel{\sqrt{3}}}{2} = 18\cancel{\sqrt{3}}$

$\Rightarrow x^2 = 12$

$\Rightarrow x = \sqrt{12} = \boxed{2\sqrt{3}}$

b) cosine rule : $a^2 = b^2 + c^2 - 2bc\cos A$

$\Rightarrow BC^2 = (2x)^2 + (3x)^2 - 2(2x)(3x)\cos 60$

$\Rightarrow BC^2 = 13x^2 - 12x^2\left(\frac{1}{2}\right)$

Question 6 continued

$\Rightarrow BC^2 = 7x^2 = 7(12)$

$\Rightarrow BC = \sqrt{7(12)} = \boxed{2\sqrt{21}}$

(Total for Question 6 is 6 marks)

7. The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote. **(3)**

The line l has equation $y = -2x + 5$

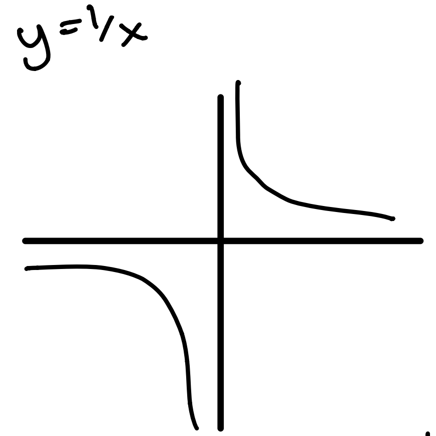
(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

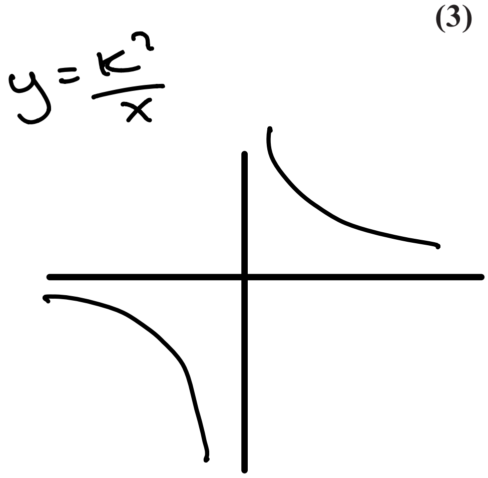
$$2x^2 - 4x + k^2 = 0$$

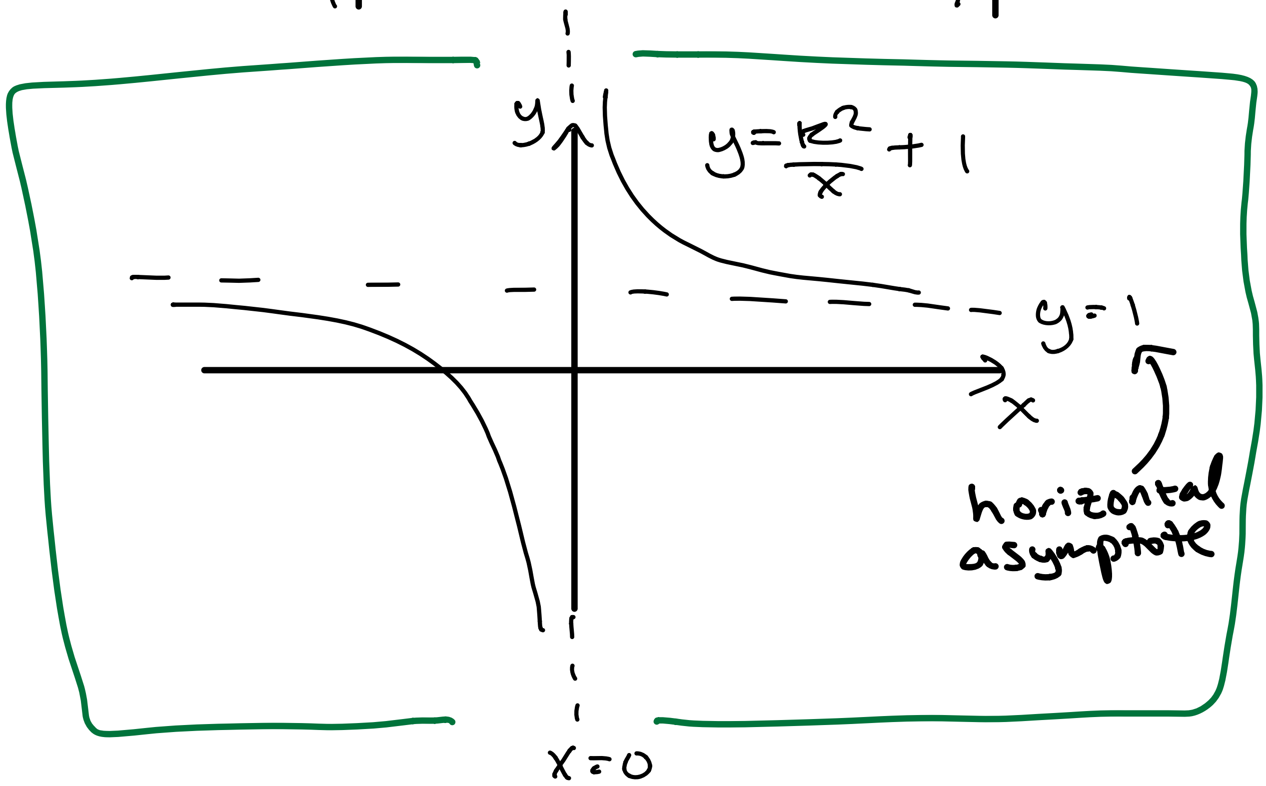
(2)

(c) Hence find the exact values of k for which l is a tangent to C. **(3)**

a)







Question 7 continued

b) $y = \frac{k^2}{x} + 1$ and $y = -2x + 5$

$\Rightarrow \frac{k^2}{x} + 1 = -2x + 5$

$\Rightarrow k^2 + x = -2x^2 + 5x$

$\Rightarrow 2x^2 - 4x + k^2 = 0$ //

c) tangent ; L will only meet C at one point.

So $b^2 - 4ac = 0$

$\Rightarrow (-4)^2 - 4(2)(k^2) = 0$

$\Rightarrow 16 = 8k^2$

$\Rightarrow 2 = k^2$

$\Rightarrow \boxed{k = \pm\sqrt{2}}$

(Total for Question 7 is 8 marks)

8. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 + \frac{3x}{4}\right)^6$$

giving each term in its simplest form.

(4)

(b) Explain how you could use your expansion to estimate the value of 1.925^6
You do not need to perform the calculation.

(1)

a) $\left(2+\frac{3x}{4}\right)^6 \simeq 2^6 + \binom{6}{1}(2)^5\left(\frac{3x}{4}\right)^1 + \binom{6}{2}(2)^4\left(\frac{3x}{4}\right)^2$

$\simeq \boxed{64 + 144x + 135x^2}$

b) solve $2+\frac{3x}{4} = 1.925$

and substitute the solution into our part (a) answer.

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9. A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where T tonnes is the total mass of tin mined in the n years after the start of mining.

Using this model,

(a) calculate the mass of tin that will be mined up to 1st January 2020,

(1)

(b) deduce the maximum total mass of tin that could be mined,

(1)

(c) calculate the mass of tin that will be mined in 2023.

(2)

(d) State, giving reasons, the limitation on the values of n.

(2)

a) $\underline{n=1}$: $T = 1200 - 3(1-20)^2$

$= \boxed{117}$ tonnes

b) 1200. (when $T = 20$ the 2nd term disappears)

c) $T_5 - T_4 = [1200 - 3(5-20)^2] - [1200 - 3(4-20)^2]$

$= \boxed{93}$ tonnes

d) As observed in part (b), Tmax is reached when $n = 20$, hence the model is only valid for $n \leq 20$.

The total mass of tin cannot decrease.

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10. A circle C has equation

$$x^2 + y^2 - 4x + 8y - 8 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The straight line with equation $x = k$, where k is a constant, is a tangent to C.

(b) Find the possible values for k.

(2)

a) $x^2 - 4x + y^2 + 8y = 8$

$(x-2)^2 - 4 + (y+4)^2 - 16 = 8$

$(x-2)^2 + (y+4)^2 = 28$ //

so centre $(2, -4)$

radius $\sqrt{28} = 2\sqrt{7}$

b) $\underline{x=k}$: $(k-2)^2 + (y^2+8y+16) = 28$

$y^2 + 8y + \left((k-2)^2 - 12\right) = 0$

$x=k$ is a tangent ; ie there is only one intersection.

so $b^2 - 4ac = 0$

$(8)^2 - 4(1)\left((k-2)^2 - 12\right) = 0$

$16 = (k-2)^2 - 12$

$28 = (k-2)^2$

Question 10 continued

$\therefore \quad k - 2 = \pm 2\sqrt{7}$

$\boxed{k = 2 \pm 2\sqrt{7}}$

(Total for Question 10 is 5 marks)

11. $$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

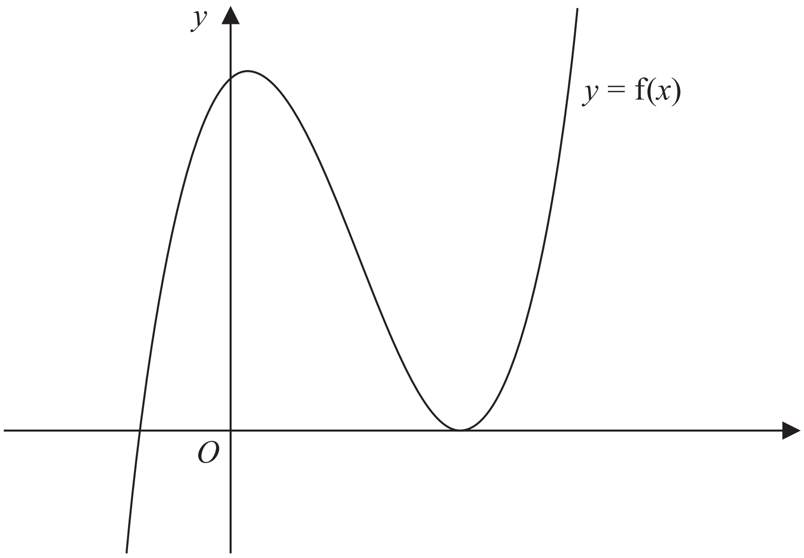


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

a) $f(4) = 2(4)^3 - 13(4)^2 + 8(4) + 48$

$= 0$ // $\therefore$ $(x-4)$ is a factor

b) long division:

$$\begin{array}{r|l} & 2x^2 - 5x - 12 \\ x-4 & 2x^3 - 13x^2 + 8x + 48 \\ & 2x^3 - 8x^2 \\ & 0 \quad -5x^2 + 8x \\ & \quad -5x^2 + 20x \end{array}$$

Question 11 continued

$$\begin{array}{r} \overline{0 \quad -12x + 48} \\ -12x + 48 \\ \hline 0 \quad 0 \end{array}$$

//

so $f(x) = (x-4)(2x^2-5x-12)$

$= (x-4)(2x+3)(x-4)$

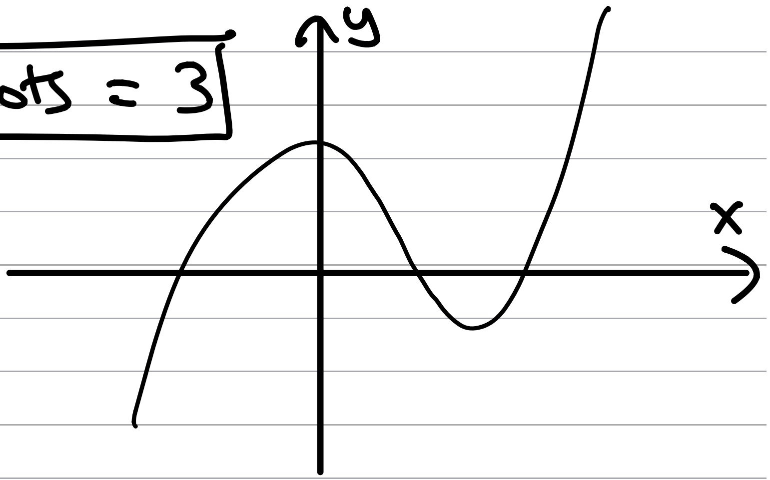
$= (x-4)^2(2x+3)$ //

hence $f(x)$ has just 2 distinct roots; $x=4$

$x=-3/2$

c) The curve is shifted down by 2 units

so $\boxed{\# \text{ of roots} = 3}$



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Question 11 continued

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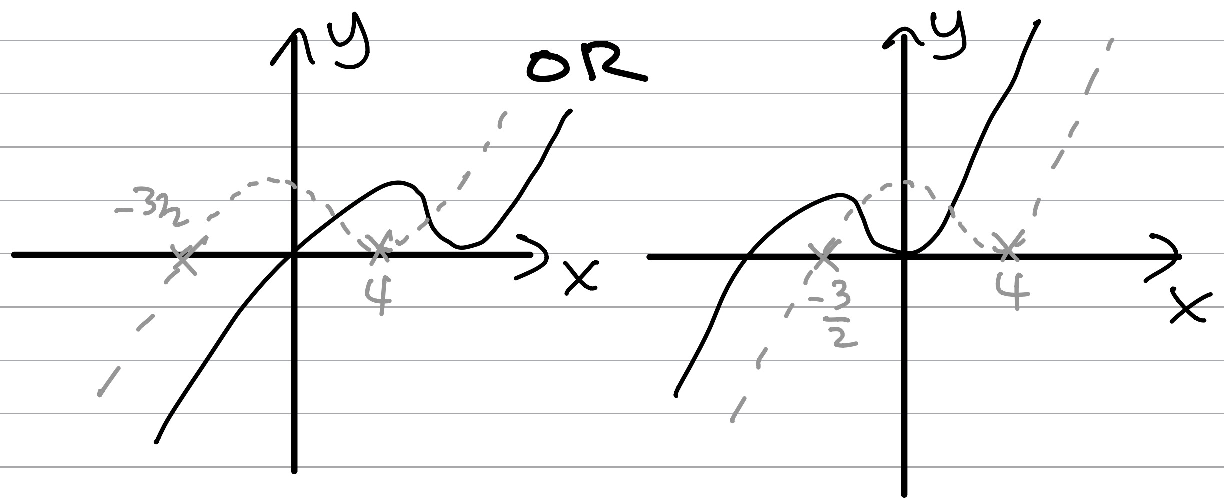
d) we could have :



f(x) shifted to the right by $3/2$

ie $f(x-3/2)$

so $\boxed{k = -\frac{3}{2}}$

f(x) shifted to the left by 4

ie $f(x+4)$

so $\boxed{k = 4}$

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12. (a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant x < 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

a) $LHS = \dfrac{10(1-\cos^2\theta) - 7\cos\theta + 2}{3 + 2\cos\theta}$

$= \dfrac{12 - 10\cos^2\theta - 7\cos\theta}{3 + 2\cos\theta}$

$= \dfrac{-[10\cos^2\theta + 7\cos\theta - 12]}{3 + 2\cos\theta}$

$= \dfrac{-[(5\cos\theta - 4)\cancel{(2\cos\theta + 3)}]}{\cancel{(3 + 2\cos\theta)}}$

$= 4 - 5\cos\theta = RHS$

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Question 12 continued

b) we can instead solve :

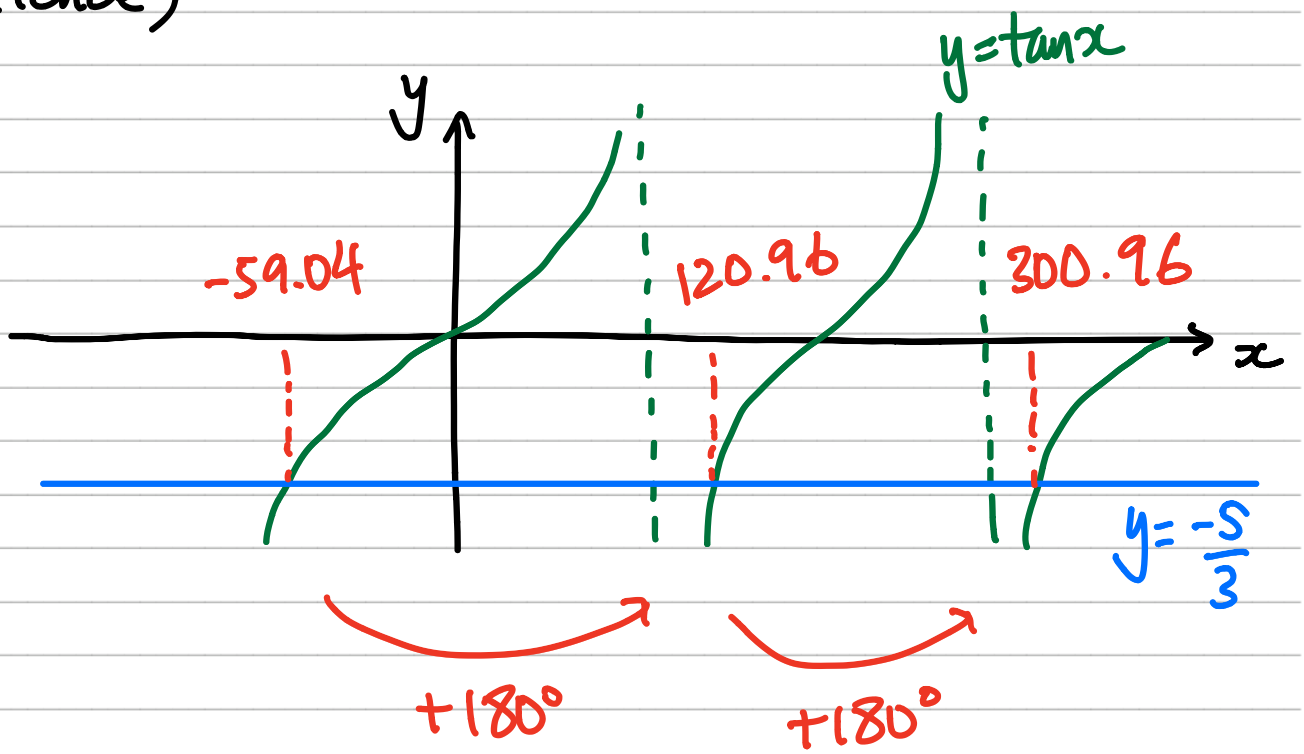
$$\cancel{4} - 5\cos x = \cancel{4} + 3\sin x$$

$$\sin x = -\frac{5}{3}\cos x$$

$\div \cos x$: $\tan x = -\frac{5}{3}$

$$x = \tan^{-1}\left(-\frac{5}{3}\right) = -59.04°$$

Hence,



$\therefore x = \underline{\underline{121°}}, \underline{\underline{301°}}$

13.

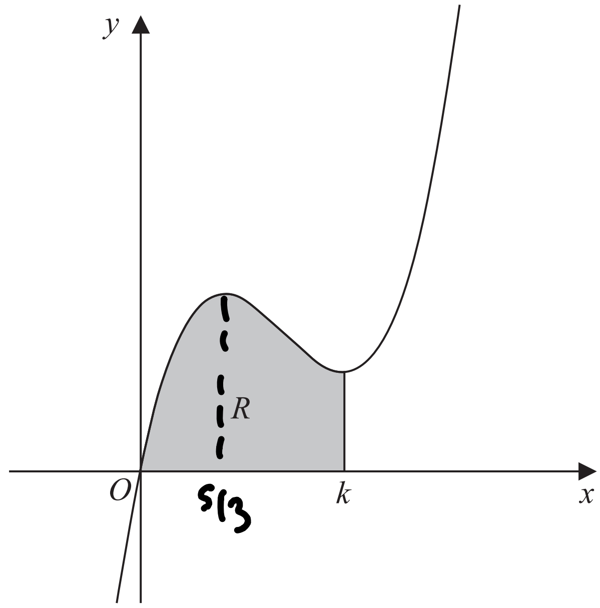


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\dfrac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

a) $\frac{dy}{dx} = 6x^2 - 34x + 40 = 0$

$(3x - 5)(2x - 8) = 0$

$x = \frac{5}{3}$ $\quad x = 4$

$x = \frac{5}{3}$ corresponds to the 1st turning point, so $x = 4$ is the one we are after. ie $k = 4$.

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Question 13 continued

$$\text{Area}_R = \int_0^4 [2x^3 - 17x^2 + 40x]\, dx$$

$$= \left[2x^4/4 - 17x^3/3 + 20x^2\right]_0^4$$

$$= \left[\tfrac{1}{2}(256) - \tfrac{17}{3}(64) + 20(16)\right]$$

$$= \left[128 - \frac{1088}{3} + 320\right]$$

$$= \left[448 - \frac{1088}{3}\right]$$

$$= \boxed{\frac{256}{3}}$$

Question 13 continued

(Total for Question 13 is 7 marks)

14. The value of a car, £V, can be modelled by the equation

$$V = 15\,700e^{-0.25t} + 2300 \qquad t \in \mathbb{R},\ t \geqslant 0$$

where the age of the car is t years.

Using the model,

(a) find the initial value of the car.

(1)

Given the model predicts that the value of the car is decreasing at a rate of £500 per year at the instant when $t = T$,

(b) (i) show that

$$3925e^{-0.25T} = 500$$

(ii) Hence find the age of the car at this instant, giving your answer in years and months to the nearest month.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

The model predicts that the value of the car approaches, but does not fall below, £A.

(c) State the value of A.

(1)

(d) State a limitation of this model.

(1)

a) $\underline{t=0}$: $V = 15700e^{0} + 2300$

$= \boxed{£18000}$

bi) at $t = T$, $\frac{dV}{dt} = -500$

$\frac{dV}{dt} = (-0.25)15700e^{-0.25t}$

$= -3925e^{-0.25t}$ //

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Question 14 continued

at $t = T$: $-3925e^{-0.25T} = -500$

$\times -1$

$\Rightarrow \quad 3925e^{-0.25T} = 500$ //

ii) $e^{-0.25T} = \frac{500}{3925} = \frac{20}{157}$

$\therefore \ln\left[e^{-0.25T}\right] = \ln\left[\frac{20}{157}\right]$

$-0.25T = \ln\left[\frac{20}{157}\right]$

$T = -4\ln\left[\frac{20}{157}\right]$

$T = 8.24$ years //

$\boxed{8 \text{ years, } 3 \text{ months}}$

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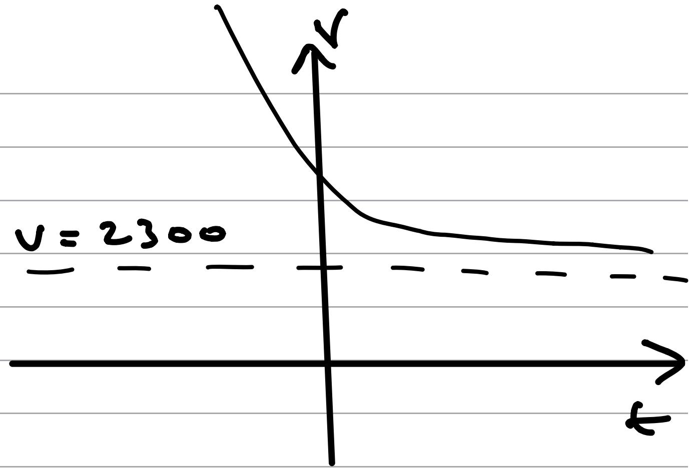
Question 14 continued

c) $\boxed{£2300}$



d) For large values of t the car is likely to be worth less than £2300, eventually.

15. Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

(4)

If n is even, then

let $n = 2K$.

$$\frac{n^3+2}{8} = \frac{(2K)^3+2}{8}$$

$$= \frac{8K^3+2}{8}$$

$$= K^3 + \frac{1}{4}$$

This is evidently not a whole number and hence not divisible by 8.

If n is odd, then

let $n = 2K+1$

$$\frac{n^3+2}{8} = \frac{(2K+1)^3+2}{8}$$

$$= \frac{(2K+1)(4K^2+4K+1) \quad +2}{8}$$

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Question 15 continued

$$= \frac{8k^3 + 12k^2 + 6k + 3}{8}$$

$$= \frac{2(4k^3 + 6k^2 + 3k) + 3}{8}$$

The numerator is an odd number because we have an even number $+3$, hence odd. Hence this is not divisible by 8 either, and thus for $n \in \mathbb{N}$, $n^3 + 2$ is <u>NOT</u> divisible by 8. //

(Total for Question 15 is 4 marks)

16. (i) Two non-zero vectors, **a** and **b**, are such that

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$$

Explain, geometrically, the significance of this statement.

(1)

(ii) Two different vectors, **m** and **n**, are such that $|\mathbf{m}| = 3$ and $|\mathbf{m} - \mathbf{n}| = 6$
The angle between vector **m** and vector **n** is 30°

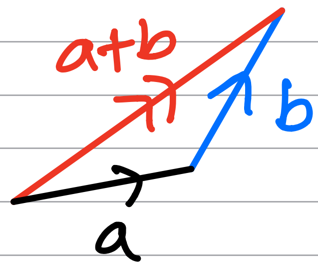
Find the angle between vector **m** and vector **m** – **n**, giving your answer, in degrees, to one decimal place.

(4)

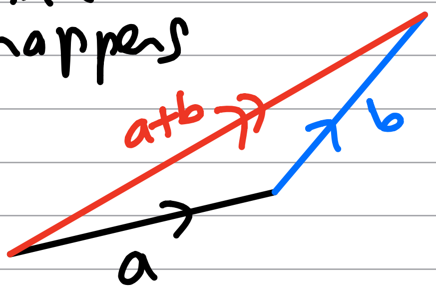
The statement says:

i) The length of the sum of 2 vectors is equal to their individual lengths summed together

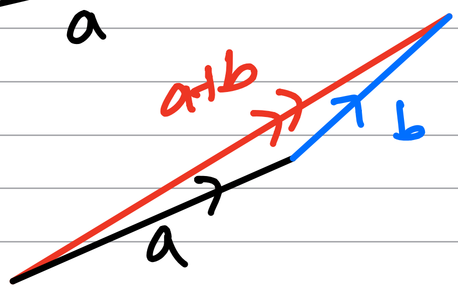
So a and b are vectors that lie on the same straight line (so parallel)



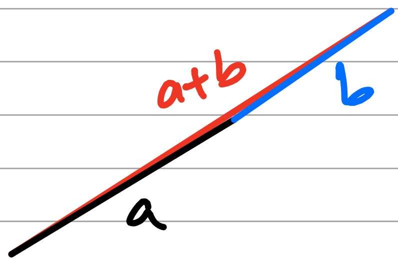
It helps to thinu about what happens as we vary a and b.



here length of a+b is less than (length a) + (length b)



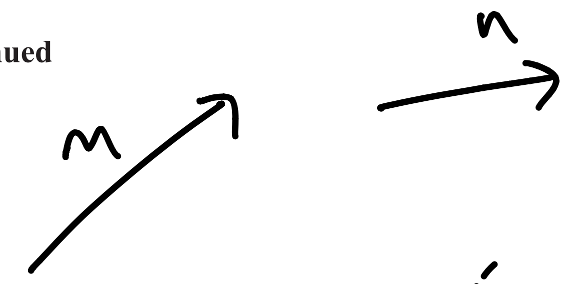
Same here, but |a|+|b| is close to |a+b|.



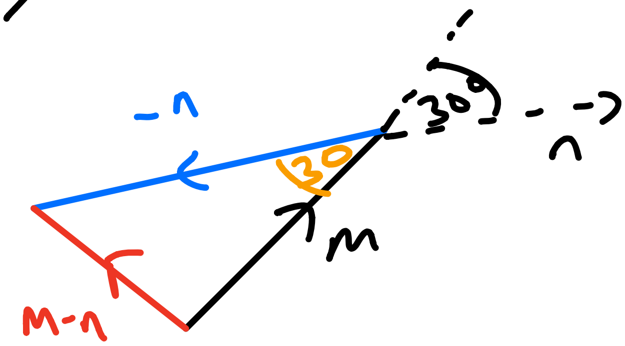
now |a+b| = |a| + |b|
all the sides lie on the same line.

Question 16 continued

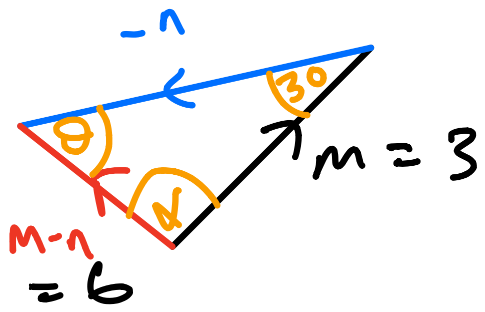
let :



then



$\Rightarrow$



angle required = α

sine rule : to find θ

$$\frac{\sin\theta}{3} = \frac{\sin 30}{6}$$

$$\therefore \theta = 14.5^\circ$$

$$\therefore \alpha = 180 - 30 - 14.5$$

$$= \boxed{135.5^\circ}$$